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AN ANALYTICAL INVESTIGATION OF THE DIRECT MEASUREMENT METHOD OF ESTIMATING THE ACOUSTIC IMPEDANCE OF A TIME-VARYING SOURCE

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This paper presents an analytical investigation of the direct method of measurement of the source impedance of a linear time-variant source. The direct method yields a frequency-dependent effective source impedance which is routinely used in a time-invariant analysis to determine the insertion loss of two different acoustic loads applied to the same source. In such an analysis the strength of the source is assumed to be invariant with load. It is shown here that there is generally no precise correspondence between the effective source impedance as given by the direct method and the characteristics of the actual source. Furthermore, it is shown that the effective source impedance values given by the direct method are functions of the acoustic load and the location of the injected signal as used in the measurement. However, the effective source resistance is always found to be positive, in accordance with experimental measurements. In this regard the direct method is an improvement on the indirect method, where physically implausible negative resistance values are often found. Finally, it is shown that the effective impedance values as given by the direct method when used with a time-invariant analysis give rise to very accurate predictions of insertion loss, even when the strength of the actual time-variant source is allowed to vary with the acoustic load.

1. INTRODUCTION

The context of this paper is the same as that of an earlier paper which investigated the indirect method [1], namely linear frequency-domain modelling of the acoustic characteristics of the exhaust or intake silencer system of an internal combustion (IC) engine [2]. Relative to other techniques, linear frequency-domain modelling is very quick and enables one to make realistic representations of the complex internal geometry which is typical of a commercial silencer. The main drawback of the technique is the difficulty in accurate characterization of the source, which is both non-linear and time variant. A schematic of the problem for the exhaust side of the engine is shown in Figure 1(a), where everything downstream of the exhaust valves is regarded as an acoustic load, Z_l on the engine. The equivalent electrical network for a single frequency component of the linear acoustic, time-invariant representation of the engine exhaust model is shown in Figure 1(b).

In order to predict insertion loss from the time-invariant analysis, namely the difference in radiated noise levels between two different systems given the same invariant source, it is necessary to know the source impedance Z_s for each frequency ω_n. Source impedance is a complex quantity, consisting of a resistance and a reactance. It characterizes not only the form of the discharge of the source into the system, but also the manner in which acoustic

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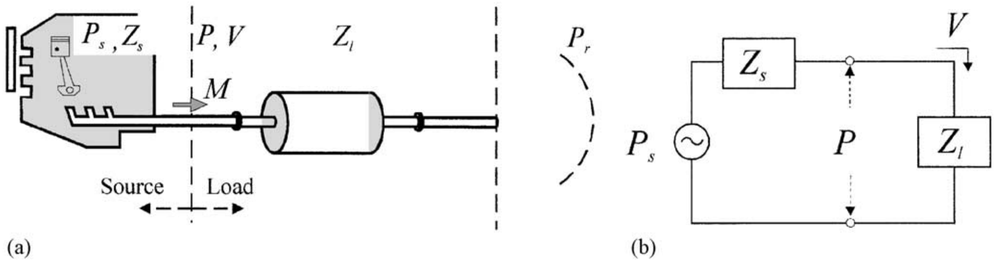

Figure 1. Source–load model: (a) acoustic source–load system; (b) electro-acoustic circuit.

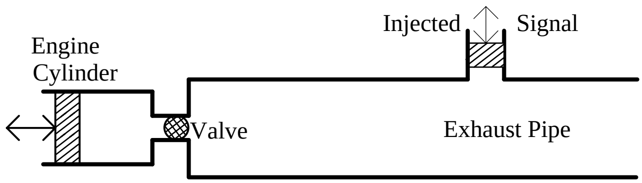

Figure 2. System for direct method of source measurement.

waves travelling towards the source are reflected by the source. In order to predict the absolute level of radiated noise from a single system with a given source, it is also necessary to know the source strength P_s for each frequency ω_n. The source data is generally obtained by experimentation, using either a direct or an indirect method.

Indirect methods [3–8] are generally used for IC-engine sources, and have the benefit of yielding both the strength and impedance of the source. Indirect methods make use of measurements relating to two or more different loads on the same source. It is normal to use simple free-field pressure measurements for four or more loads. However, it has been shown [1] that the effective source characteristics of a time-variant source as measured by the indirect method have no physical meaning. This finding is in agreement with the fact that the resistance of time-varying sources as measured by the indirect method is frequently found to be negative, which is physically implausible.

In contrast, the direct method [9–14] has always been found to give positive resistance values for various source types. Thus, one might surmise that, since the measured values are physically plausible, the method might actually yield a meaningful value of impedance for a time-varying source. Unfortunately, the direct method does have some disadvantages. Firstly, it only yields a value for the source impedance. The source strength is not given. Secondly, the direct method requires the use of an injected signal, placed somewhere in the load section, see Figure 2. The magnitude of the injected signal is generally chosen to be so high as to make the output of the primary source negligible by comparison. The impedance of the effectively passive source termination is then measured by conventional techniques. Alternatively, sophisticated signal processing is required to extract the source component from an overall measurement, such that the impedance can be obtained from the component of the injected signal alone. The injected signal must be at least comparable in magnitude to the source signal for measurement errors to remain tolerable in this process. For an IC-engine source, it is a major problem to provide an injected signal which is dominant over, or even of comparable magnitude to, the source signal and, even if this is possible, the resultant sound level will be so high as to make non-linear effects significant. Furthermore, the method requires in-duct measurements, and the exhaust side of an IC-engine represents a hostile environment for delicate microphones. Despite these difficulties, given the failure of the indirect method to yield physically meaningful source properties, it seems appropriate to at least investigate whether in principle the direct method is any better.

In this paper, an analytical investigation is made of the discharge from an idealized linear, time-variant source, to determine whether the source impedance as given by the direct method bears any relation to the actual properties of the source. Specifically, as in the earlier paper [1] for the indirect method, it is not the intention to accurately model an IC-engine source, which is also non-linear. The source is made time-variant by allowing the discharge to occur through a valve whose open area is time-dependent, the valve being considered as part of the source. In particular, the valve is assumed to be completely closed over part of the cycle, as is the case for an IC-engine. A simple idealized linear equation of discharge through the valve is assumed. Since the purpose is to determine the relationship between the actual characteristics of a given linear time-variant source and the source characteristics that would be determined if that same source were assumed to be both linear and time-invariant, it does not matter that the actual source used is not a realistic representation of an IC-engine.

Section 2 of the paper presents a simple inertial model of the acoustic flow through a valve. This model is identical with that used in the earlier analysis of the indirect method [1]. The analysis of the system is slightly altered, however, to allow for an injected sound at a different frequency to that of any of the harmonics of the valve motion. It is shown that the frequency-dependent source impedance, as used in a time-invariant analysis, can only represent the characteristics of the actual source correctly when the latter is also time-invariant. Section 3 gives a theoretical analysis of the direct method of measurement of impedance of the actual time-variant source, as introduced in section 2. It is shown that in theory the effective source impedance values given by the direct method are functions of the acoustic load and the location of the injected signal as used in the measurement.

Section 4 gives some comparisons between the actual source conditions and the effective source admittance or impedance as given by the direct method. The precise form of valve motion must be specified for such results. An actual time-variant source gives rise to an impedance matrix for a single operating condition of the engine. It is found that the matrix is not diagonally dominant, and hence the diagonal coefficients cannot be used as an accurate approximation for the impedance values in a time-invariant analysis. However, the effective source resistance is always found to be positive, in accordance with experimental measurements. Furthermore, it is shown that, in practice, the location of the injected signal has no significant influence upon the impedance results from the direct method, and the influence of the acoustic load is also quite weak, except at specific harmonics of the valve motion.

Finally, in section 5, examples are given to determine the errors in insertion loss results when one uses a time-invariant analysis with effective source impedance values given by the direct method, when the actual source is time-variant. It is shown that the errors are insignificant. This statement is shown to hold true both at harmonic frequencies of the actual engine source, and at intermediate frequencies where an assumed secondary source within the exhaust duct provides the noise. Furthermore, the errors are even insignificant when the pressure within the source region and the volume velocity through the exhaust valve are allowed to vary with acoustic load, in violation of further assumptions inherent in the time-invariant analysis.

2. ADMITTANCE OF A LINEAR INERTIAL SOURCE

With reference to Figure 3, let the pressures on the upstream and downstream sides of the valve be $P_s(t)$ and $P(t)$, respectively, and let the velocity of flow through the valve be $u(t)$.

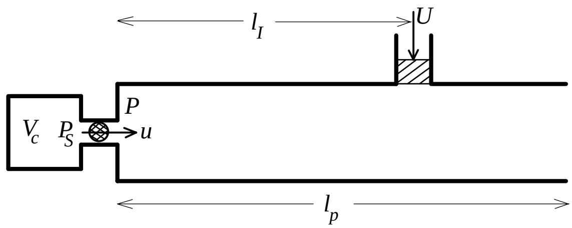

Figure 3. System for analysis of the direct method.

Both pressures are assumed as relative to atmospheric pressure. Following [1], an appropriate linear expression for the time-varying flow through the valve, which is indeed accurate [15] for flow velocities below 10 [m/s], is given by

$$[P_s(t) - P(t)] = \frac{\bar{\rho}\bar{c}u(t)}{C_D} + \bar{\rho}l\,\frac{\mathrm{d}u(t)}{\mathrm{d}t}, \tag{1}$$

where C_D is a constant non-dimensional discharge coefficient and $\bar{\rho}$, $\bar{c}$ are the mean density and speed of sound of the gas flow through the valve respectively. The variable l is the thickness of the orifice including the mass end corrections. The time-variant nature of the source is introduced by letting the valve area $\tilde{A}(t)$ vary periodically with time, with period T. In particular, the valve is assumed to be closed over some portion of the cycle, at which time the velocity of discharge will be zero. Thus, from equation (1), the equation of discharge throughout a cycle can be written as

$$\bar{\rho}\bar{c}\left[1 + \frac{lC_D}{\bar{c}}\frac{\mathrm{d}}{\mathrm{d}t}\right]V(t) = C_D\tilde{A}(t)\,[P_s(t) - P(t)], \quad \tilde{A}(t)\left\{\begin{matrix} \neq 0, & 0 \leqslant t \leqslant \tau \\ = 0, & \tau \;\, t \leqslant t \leqslant T \end{matrix}\right\}, \tag{2}$$

where $V(t)$ is the volume velocity of the flow through the valve. Let $A(t) = \tilde{A}(t)/A_{max}$ be a non-dimensional valve area, where A_{max} is the maximum open area of the valve at any time, and let $v(t) = (\bar{\rho}\bar{c}/A_p)V(t)$, where A_P is the constant area of the pipe into which the source exhausts. Equation (2) can then be re-written as

$$\left[\frac{1}{C_D} + \frac{l}{\bar{c}}\frac{\mathrm{d}}{\mathrm{d}t}\right]v(t) = CA(t)\,[P_s(t) - P(t)], \quad \left\{\begin{matrix} 0 < A(t) \leqslant 1, & 0 \leqslant t \leqslant \tau \\ A(t) = 0, & \tau \leqslant t \leqslant T \end{matrix}\right\}, \tag{3}$$

where $C = A_{max}/A_P$. Let the injected signal be harmonic with frequency Ω. Now the period of the cyclic valve motion is T, thus the valve open area can be written as a sum of Fourier components of frequencies $\omega_j = 2\pi j/T$. It follows from equation (3) that the acoustic pressure and velocity within the system must have frequency components $\Omega + \omega_j$. Thus the variables can each be expanded as complex Fourier series:

$$v(t) = \sum_{j=-\infty}^{j=+\infty} v_j\,\mathrm{e}^{\mathrm{i}(\Omega+\omega_j)t}, \quad P_s(t) = \sum_{j=-\infty}^{j=+\infty} S_j\,\mathrm{e}^{\mathrm{i}(\Omega+\omega_j)t},$$

$$P(t) = \sum_{j=-\infty}^{j=+\infty} P_j\,\mathrm{e}^{\mathrm{i}(\Omega+\omega_j)t}, \quad A(t) = \sum_{j=-\infty}^{j=+\infty} A_j\,\mathrm{e}^{\mathrm{i}\omega_j t}. \tag{4a–d}$$

In particular, it should be noted that the $j = 0$ components refer to frequency component Ω, that of the injected signal, not steady state conditions. Fourier expansion of all variables, as in equation (4), followed by a restriction to the lowest N harmonics leads to the finite matrix equation

$$\begin{bmatrix} D_{-N} & 0 & 0 & 0 & 0 \\ 0 & D_{-j} & 0 & 0 & 0 \\ 0 & 0 & D_0 & & 0 \\ 0 & 0 & 0 & D_j & 0 \\ 0 & 0 & 0 & 0 & D_N \end{bmatrix} \begin{bmatrix} v_{-N} \\ \cdots \\ v_0 \\ \cdots \\ v_N \end{bmatrix} = \begin{bmatrix} A_0 & \cdots & A_{-N} & \cdots & A_{-2N} \\ \cdots & A_0 & \cdots & \cdots & \cdots \\ A_N & \cdots & A_0 & \cdots & A_{-N} \\ \cdots & \cdots & \cdots & A_0 & \cdots \\ A_{2N} & \cdots & A_N & \cdots & A_0 \end{bmatrix} \begin{bmatrix} S_{-N} - P_{-N} \\ \cdots \\ S_0 - P_0 \\ \cdots \\ S_N - P_N \end{bmatrix} \quad (5)$$

or

$$[\mathbf{D}]\{\mathbf{v}\} = [\mathbf{A}](\{\mathbf{S}\} - \{\mathbf{P}\}), \quad (6)$$

where $D_j = (1/C_D + \mathrm{i}k_j l)/C$ and $k_j = (\Omega + \omega_j)/\bar{c}$. The coefficients of the admittance matrix $[\mathbf{D}]^{-1}[\mathbf{A}]$ and hence the impedance matrix $[\mathbf{A}]^{-1}[\mathbf{D}]$ of the time-varying source follow simply from knowledge of the geometry and motion of the valve. However, it was noted earlier [1] that if the valve is completely shut during any instant of its cycle, then matrix $[\mathbf{A}]$ becomes singular as the number of modes increases—$[\mathbf{A}]^{-1}$ does not exist. Thus in the analytical development which follows only the admittance matrix will be employed, not its inverse, and general comparisons will be made in terms of admittance rather than impedance.

The conventional time-invariant source model as represented by Figure 1(b) is characterized by the equation

$$P_s(\Omega) - P(\Omega) = Z_s(\Omega)V(\Omega) = \zeta_s(\Omega)v(\Omega) \quad (7)$$

or

$$v(\Omega) = \alpha_s(\Omega)[P_s(\Omega) - P(\Omega)] \quad (8)$$

where ζ_s is the non-dimensional source impedance for any frequency component Ω and α_s it its inverse, the source admittance. Comparison of equations (6) and (8) indicates that the time-variant source has an admittance matrix $[\mathbf{D}]^{-1}[\mathbf{A}]$ and that the two equations can only be equivalent if this admittance matrix is diagonal, which from equation (6) implies that the valve area $A(t) = A_0$, a constant. Thus if the source has any time-variance, there can be no equivalence between time-variant and time-invariant representations of the source, as is only to be expected.

3. ANALYSIS OF THE DIRECT METHOD

The direct method of measurement of source impedance measures the impedance at entry to the source region, as viewed from the exhaust side of the system, at the frequency of the injected signal, Ω. Thus the "measured" source admittance $\alpha_m(\Omega) = -v_0/P_0$. It follows from equation (6) that

$$\alpha_m(\Omega) = \sum_{j=-N}^{+N} \frac{A_{-j}}{D_0}(P_j - S_j)/P_0. \quad (9)$$

Unlike the actual admittance matrix of the source, which is dependent only on the source geometry and valve motion, the measured admittance also depends upon the relative magnitude of the components of the acoustic pressure within the system. These in turn are dependent upon the precise load connected to the source. The only exception to the previous statement occurs when $A_j = 0$, $j \neq 0$, i.e., the open area of the valve is constant and the source is time invariant. It then follows from equations (8) and (9) that the measured admittance $\alpha_m(\Omega)$ is equivalent to the required admittance $\alpha_s(\Omega)$ provided that $P_s(\Omega) = S_0 = 0$, in which case $\alpha_m = \alpha_s = A_0/D_0$.

Thus even in the time-invariant case, when the valve does not move, there must be zero pressure fluctuation within the source region for the direct method to yield the required source admittance. The reason for this is that the "measured" source admittance is actually the input admittance to a passive source region, $\alpha_m(\Omega) = -v_0/P_0$, whereas the required admittance of the active source follows from equation (8) and is $\alpha_s(\Omega) = -v_0/(P_0 - S_0)$. In the context of the simple source used here, the former is the input admittance to a Helmholtz resonator formed from the "cavity" of the source region and the "neck" of the valve constriction, while the latter is simply the admittance of the valve alone. The two are identical only if the injected signal itself does not cause any significant pressure fluctuation within the source region, thereby influencing S_0. This will only be true either if the valve has a vanishingly small admittance, i.e., it is effectively shut, or else if the source "cavity" has infinite admittance, i.e., it is extremely large such that the mass influx/efflux from the injected signal does not alter the pressure in the source region.

Consider again the general analysis of the full system shown in Figure 3. Since the engine source must have negligible output as compared with the injected signal for the direct method to be applicable, then the piston motion in the engine cylinder must be of negligible amplitude as compared to the amplitude of the injected signal. Likewise any in-cylinder source pressure variation due to explosions of the fuel and air mixture, etc. must be negligible as compared to the pressure fluctuations in the system caused by the injected signal. Since the velocity of motion of the piston source, $U_C(t)$, is considered to be negligibly small in comparison to the velocity field induced by the injected signal, of velocity $U(t)$, the source must effectively be a constant volume region and can be modelled as such.

With restriction to low frequencies, a lumped-mass model of the effectively constant volume source, of volume V_c, can be used to give the coefficients, c_j of the diagonal matrix $[\mathbf{C}]$, where

$$\{\mathbf{S}\} = [\mathbf{C}]\{\mathbf{v}\}, \quad c_j = \mathrm{i}A_P/k_jV_c. \tag{10a, b}$$

It can be seen that as the source volume V_c becomes large, the coefficients c_j tend to zero and hence so to do the pressure fluctuations S_j in the source region, as noted earlier. However, the cylinder of an IC-engine does not represent such a large volume source.

The precise location of the injected signal in the overall system is governed by lengths l_P, that of the complete exhaust pipe, and l_I, the distance from the valve to the injected signal; see Figure 3. At frequencies other than that of the injected signal, corresponding to $j = 0$, the conventional four-pole transfer matrix [2] of a uniform pipe can be used to relate the acoustic pressure and volume velocity of the jth harmonic at the source outlet to the same properties at the tailpipe orifice, say plane R, as

$$\begin{Bmatrix} P \\ v \end{Bmatrix} = \begin{bmatrix} \cos k_jl_P & \mathrm{i}\sin k_jl_P \\ \mathrm{i}\sin k_jl_P & \cos k_jl_P \end{bmatrix} \begin{Bmatrix} p_R \\ v_R \end{Bmatrix}. \tag{11}$$

If the known radiation impedance of the tailpipe orifice is $\zeta_R = p_R/v_R$, it follows from equation (11) that

$$P = \left(\frac{\zeta_R \cos k_j l_P + \mathrm{i} \sin k_j l_P}{\mathrm{i}\zeta_R \sin k_j l_P + \cos k_j l_P}\right) v, \quad j \neq 0. \tag{12}$$

For component $j = 0$, one must include the effect of the injected signal. A uniform pipe transfer matrix can be used to relate the acoustic properties between the source outlet and the inlet to the plane of the injected signal, say plane $1I$, as

$$\begin{Bmatrix} P \\ v \end{Bmatrix} = \begin{bmatrix} \cos k_0 l_I & \mathrm{i} \sin k_0 l_I \\ \mathrm{i} \sin k_0 l_I & \cos k_0 l_I \end{bmatrix} \begin{Bmatrix} p_{1I} \\ v_{1I} \end{Bmatrix} \tag{13}$$

and again between the outlet at the plane of the injected signal, say plane $2I$ and the tailpipe orifice, as

$$\begin{Bmatrix} p_{2I} \\ v_{2I} \end{Bmatrix} = \begin{bmatrix} \cos k_0(l_P - l_I) & \mathrm{i} \sin k_0(l_P - l_I) \\ \mathrm{i} \sin k_0(l_P - l_I) & \cos k_0(l_P - l_I) \end{bmatrix} \begin{Bmatrix} p_R \\ v_R \end{Bmatrix}. \tag{14}$$

Use of a constant pressure condition, $p_{1I} = p_{2I}$, and mass conservation condition, $v_{1I} + v_I = v_{2I}$, in the plane of the injected signal, together with equations (13) and (14), enables one to write an expression similar to equation (12) for the component $j = 0$, but one which now includes v_I. Thus all components can be combined in the single matrix equation

$$\{\mathbf{P}\} = [\mathbf{B}]\{\mathbf{v}\} + v_I\{\mathbf{d}\}, \tag{15}$$

where the coefficients of the diagonal matrix $[\mathbf{B}]$ and the vector $\{\mathbf{d}\}$are

$$b_j = \frac{\zeta_R \cos k_j l_P + \mathrm{i} \sin k_j l_P}{\mathrm{i}\zeta_R \sin k_j l_P + \cos k_j l_P}, \quad d_j = \begin{cases} b_0 \cos k_0 l_I - \mathrm{i} \sin k_0 l_I, & j = 0 \\ 0, & j \neq 0 \end{cases} \tag{16a, b}$$

respectively.

Equations (6), (10a) and (15) then combine to give

$$([\mathbf{D}] + [\mathbf{A}][\mathbf{B}] - [\mathbf{A}][\mathbf{C}])\{\mathbf{v}\} = -[\mathbf{A}]\{\mathbf{d}\}\, v_I. \tag{17}$$

The solution for $\{\mathbf{v}\}/v_I$ follows from this equation, and hence that for $\{\mathbf{P}\}/v_I$ is given by equation (15). The "measured" source admittance $\alpha_m(\Omega) = -v_0/P_0$ then follows simply.

4. RESULTS FOR SOURCE ADMITTANCE AND IMPEDANCE

By way of example, consider a valve whose non-dimensional open area is given by

$$A(t) = \begin{cases} \frac{1}{2}\left[1 + \cos\left(\frac{6\pi t}{T} - \pi\right)\right], & 0 \leqslant t \leqslant T/3 \\ 0, & T/3 \leqslant t \leqslant T \end{cases} \quad A(t+T) = A(t). \tag{18}$$

The valve motion is chosen to represent that of a four-stroke engine, which is generally timed to open before BDC and close after TDC, extending the open period beyond $T/4$. It is the same valve motion as used previously [1], where the profile was given and it was shown that a good approximation follows for $N = 10$. The valve is assumed to have a maximum open area of 100 mm^2 and an effective "neck" length of 10 mm including end corrections, which are incorrectly assumed to be invariant with frequency. The speed of sound throughout the valve and exhaust region is taken as 500 m/s, representative of the exhaust side of an engine. The value of the discharge coefficient C_D was evaluated as that which would give the same steady flow velocity of 10 m/s from both the linear model of equation (1) and from the precise non-linear model of the valve [15]. The value of 10 m/s was chosen as it marks the transition from inertial to resistive effects of the valve.

For a purely resistive valve flow, the coefficients of matrix [**D**] are all constant ($= 1/C_D C$), see equation (5). Therefore the admittance matrix is simply proportional to [**A**], where the coefficients of matrix [**A**] are just the Fourier coefficients of the valve open area, and follow immediately once given the valve motion. These coefficients are real and are listed in reference [1]. The diagonal elements of the matrix are constant and are slightly greater than the off-diagonal elements, but the matrix is not diagonally dominant. Some of the coefficients of the corresponding impedance matrix are also tabulated in reference [1]. They are also real, but typically of magnitude 10^5 to 10^8, and tend to infinity as N increases.

For a more realistic valve flow model which includes inertial effects, as assumed here, the coefficients of the admittance and impedance matrices are complex and vary with frequency, due to the elements of matrix [**D**]. For all frequencies, the coefficients of the admittance matrix are finite, whereas those of the impedance matrix always tend to infinity as N increases. Again, the diagonal elements of both the admittance and impedance are of slightly greater magnitude than the off-diagonal elements, but the matrices are nowhere near to being diagonally dominant. This lack of diagonal dominance implies that the effects of the frequency components do not separate and hence that the actual characteristics of the source cannot be correctly represented simply by a frequency-dependent strength and impedance.

Let the valve connect a source region of constant volume $V_c = 0{\cdot}25$ litre to a uniform pipe of length 1·5 m and radius 20 mm. The position of the injected signal is taken to be 0·9 m from the valve. Results have been obtained by using only the first 10 acoustic harmonics, i.e., $N = 10$, and a fundamental frequency for the valve motion of 50 Hz. The effectively "measured" results of source admittance and hence impedance, as discussed in section 3, can be obtained for any frequency of the injected signal. Figures 4 and 5 show the "measured" admittance and impedance results, respectively, for all frequencies up to 500 Hz. The

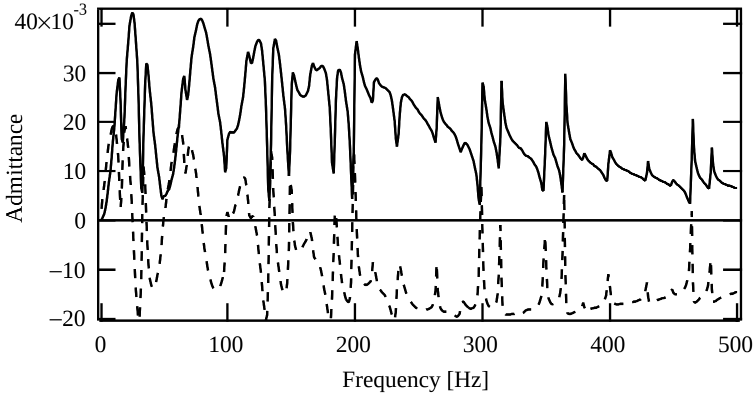

Figure 4. Source admittance as given by the direct method: $l_P = 1{\cdot}5$ m; $l_I = 0{\cdot}9$ m; ——, real part; ……, imaginary part.

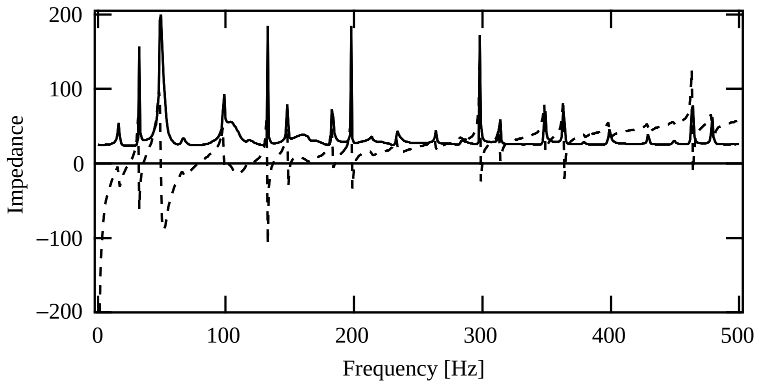

Figure 5. Source impedance as given by the direct method: $l_P = 1{\cdot}5$ m; $l_I = 0{\cdot}9$ m; ——, resistance; ……, reactance.

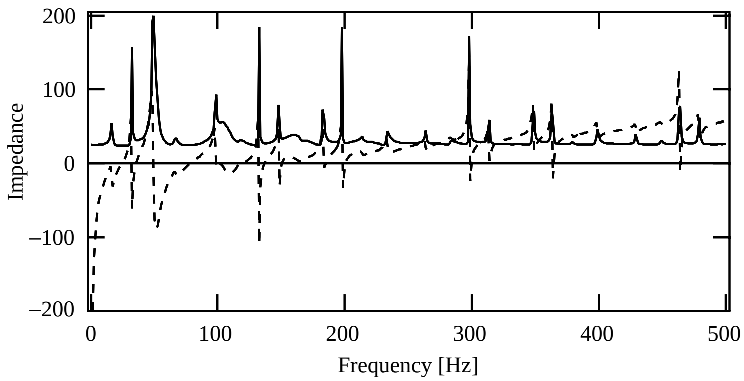

Figure 6. Source impedance as given by the direct method: $l_P = 1{\cdot}5$ m; $l_I = 0{\cdot}25$ m; ——, resistance; ……, reactance.

admittance and impedance values are seen to have peaks at or very close to most of the harmonics of the valve motion, at multiples of 50 Hz. Exceptions seem to occur when such a harmonic frequency coincides with a frequency of resonance of the exhaust pipe. The latter occur when the length of the exhaust pipe is an odd number of quarter wavelengths, which in this example is at frequencies of 83, 248 and 413 Hz. Thus, the expected peak at a valve motion harmonic of 250 Hz is cancelled completely, whereas those at frequencies of 100 and 400 Hz probably have their amplitude diminished by the exhaust pipe resonances at 83 and 413 Hz respectively. At the higher frequencies the effect of the valve harmonics is diminished anyway, since only 10 harmonics were used and the amplitude of any higher harmonics was negligible.

Figure 6 shows measured impedance results for a second example case. The only difference between this and the previous example is that the distance from the valve to the position of the injected signal has been reduced to 0·25 m in this second case. There is no discernible difference between the results shown in Figures 5 and 6. This indicates that the location of the injected signal has no significant influence on the "measured" impedance results, as one would hope. However, given that the distance l_I features in the coefficients b_j, see equation (16b), which form part of the equation set for determination of the impedance, this result was by no means assured without very detailed analysis of the system of equations.

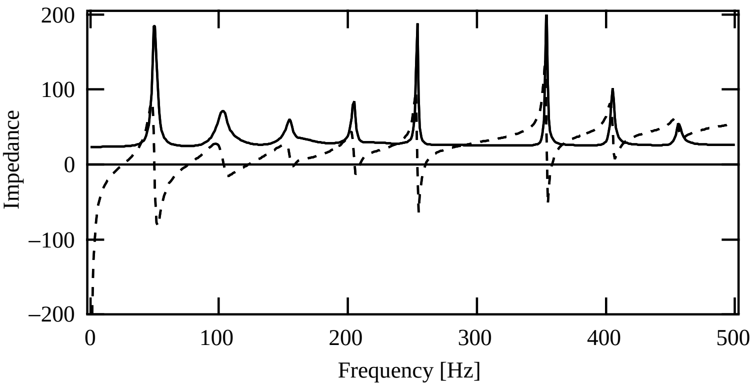

Figure 7. Source impedance as given by the direct method: $l_P = 0{\cdot}4$ m; $l_I = 0{\cdot}25$ m; ——, resistance; ......, reactance.

Figure 7 shows measured impedance results for a third example case. The only difference between this case and the second example is that the length of the exhaust pipe has now been reduced to 0·4 m. There is a very noticeable difference between the results of Figures 6 and 7. In particular, a peak at the fifth valve harmonic of 250 Hz is seen clearly in Figure 7, but the peak at the sixth harmonic of 300 Hz is now missing. This is because the first quarter-wave resonance of the 0·4 m length pipe for Figure 7 occurs at a frequency of 303 Hz and has suppressed the expected peak at 300 Hz, whereas the peak at the fifth harmonic was suppressed by the second quarter-wave resonance in Figure 6. Furthermore, all of the peaks visible in Figure 7 correspond to valve harmonics. The other peaks seen in Figures 5 and 6 have disappeared because they are a function of exhaust pipe length and the shorter length of 0·4 m used for Figure 7 has caused them to shift to higher frequencies above the range shown in the figure. The precise reason for these other peaks has not been discovered. However, a comprehensive set of test cases have shown them to be a feature only of exhaust pipe length, as well as the speed of sound, of course. In particular, the assumed volume of the source has no influence upon them, so none of them are related to Helmholtz resonances of the exhaust pipe and source volume.

The admittance and impedance values from the direct method, at the valve harmonic frequencies, should be compared with the diagonal coefficients of the actual admittance and impedance matrices. As noted earlier, since the latter are not diagonally dominant, there will not be good correlation. In particular the direct method gives high but finite impedance values of $O[10^2]$, whereas the impedance matrix has diagonal coefficients of $O[10^8]$, tending to infinity as the number of harmonics is increased. There is obviously a corresponding mismatch in admittance values. One notable feature, however, is that the resistance values from the direct method are always positive. This is in line both with what one would expect in principle, and with what has been observed experimentally from the direct method, but is in marked contrast with the indirect method [1].

In between the harmonic frequencies of the valve motion, it is seen from Figures 5–7 that the "measured" impedance is generally almost constant in value, apart from where any of the further resonance peaks are observed. Now the IC-engine source does not emit sound other than that at its firing frequency and higher harmonics thereof, corresponding to the harmonics of the valve motion. Thus, the source impedance in between such frequencies is only of relevance if there are secondary sources in the exhaust system. However, it is conventional to obtain a full spectrum of insertion loss from only a single operating condition of an IC-engine, which implies that such secondary broad-band sources do exist.

The main secondary source is probably the flow-generated noise from the turbulent outflow of the valve. Whatever the physical cause of a secondary source, its action is just like that of an injected signal, and hence the impedance of the engine source as given by the direct method is entirely appropriate. Since the impedance values are scarcely affected by acoustic load at such frequencies, one would expect the insertion loss to be given fairly accurately.

5. RESULTS FOR INSERTION LOSS

In the light of the preceding discussion, it is of interest to evaluate the insertion loss for some specific example, to give an indication of the magnitude of the error introduced into such results by assuming a time-invariant source with an impedance as given by the direct method. The actual time-variant source used for comparison, and for evaluation of the source impedance by the direct method, is the same as in section 4. The equivalent time-invariant circuit shown in Figure 1(b) can be used to evaluate insertion loss, if the source pressure is assumed not to change when two different loads are applied to the source, the so-called constant pressure model. An alternative circuit, with the same source impedance, can be used if the velocity of outflow through the valve is assumed not to change when two different loads are applied to the source, the so-called constant velocity model. Identical insertion loss results follow from either case. However, with reference to Figure 2, a more realistic assumption is that the piston velocity of the source does not change when two different loads are applied to the source, in which case neither of the two preceding conditions is precisely met.

In the first instance, the pressure in the source region will be assumed to remain invariant with acoustic load. Although this is unrealistic, it represents the most ideal case for comparison with the time-invariant analysis. For any given acoustic load on the source, an overall transfer matrix can be deduced [2] to relate the acoustic pressure and volume velocity of the jth harmonic at the source outlet to the same properties at the tailpipe orifice, as

$$\begin{Bmatrix} P \\ v \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} p_R \\ v_R \end{Bmatrix}, \tag{19}$$

where T_{mn} are the four-pole coefficients of the transfer matrix. Given the radiation impedance of the tailpipe orifice, $\zeta_R = p_R/v_R$, an expression for the load impedance on the source $\zeta_j = P/v$ then follows for this harmonic. Thus, from equation (6),

$$([\mathbf{D}] + [\mathbf{A}][\mathbf{Z}_L])\{\mathbf{v}\} = [\mathbf{A}]\{\mathbf{S}\}, \tag{20}$$

where the jth coefficient of the diagonal load impedance matrix $[\mathbf{Z}_\mathrm{L}]$, where $\{\mathbf{P}\} = [\mathbf{Z}_L]\{\mathbf{v}\}$, is ζ_j from above. If the source pressure vector $\{S\}$ is assumed known, equation (20) can be solved for the volume velocity vector from the source region. Hence, using the load impedance, the pressure vector downstream of the valve follows. From the transfer matrix of the overall silencer system or reference system, and the known radiation impedance, the free-field acoustic pressure can be found for either system, and hence the insertion loss follows from comparison of the two separate systems [2].

By way of example, a simple expansion chamber silencer system is considered, with downpipe length 1·1 m, expansion box length 0·7 m and tailpipe length 0·2 m. The area expansion ratio of the silencer box is taken to be 16·0. The reference system for insertion loss results is assumed to be a uniform pipe of length 2 m, of the same cross-sectional size as the downpipe and tailpipe of the silencer system. Figure 8 shows the insertion loss for these

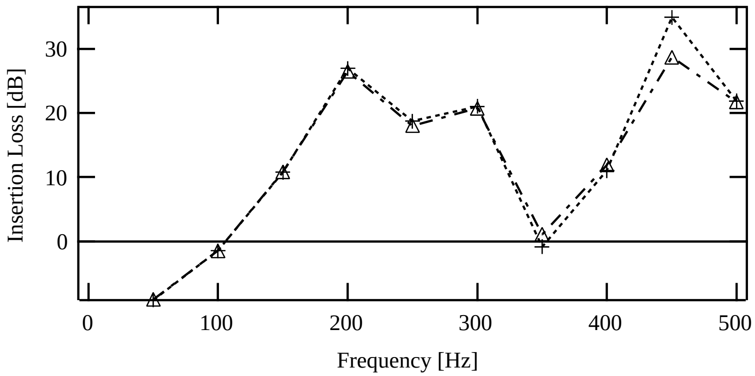

Figure 8. Insertion loss for an assumed constant pressure source: ------, + actual; ------, Δ from "measured" source impedance.

systems as given by the actual time-variant source assumed above, as compared with the insertion loss given by the time-invariant analysis if one uses the source impedance as calculated by the direct method. To be precise, the source impedance values of Figure 5 were used. There is no injected signal present now so the actual source only emits sound at harmonics of the source piston and valve motion. Thus, results are given only for these modes, even though values for all frequencies can be evaluated from the time-invariant approach. It is seen from Figure 8 that the time-invariant analysis, when using source impedance values as given by the direct method, yields very accurate results.

Next, comparison will be made when using the more realistic approximation for an actual time-variant source, namely that the volume velocity of the source piston does not vary with the acoustic load of the exhaust system. The source piston is assumed to move harmonically at the fundamental frequency of the valve motion with constant amplitude for any exhaust system. Thus, when using a lumped model of the source as earlier,

$$\{\mathbf{S}\} = [\mathbf{C}](\{\mathbf{v}\} - \{\mathbf{u}\}), \tag{21}$$

where the only non-zero components of the volume velocity vector $\{\mathbf{u}\}$ of the piston source are u_1 and u_{-1}. Substitution from this equation into equation (6) gives

$$([\mathbf{D}] - [\mathbf{A}][\mathbf{C}] + [\mathbf{A}][\mathbf{Z}_L])\{\mathbf{v}\} = -[\mathbf{A}][\mathbf{C}]\{\mathbf{u}\}, \tag{22}$$

where again the diagonal load impedance matrix $[\mathbf{Z}_L] = \{\mathbf{P}\}/\{\mathbf{v}\}$ is known for a given load. Thus, equation (22) can be solved for the volume velocity from the source region and hence eventually the insertion loss of two separate systems follows, as outlined earlier. The results, for the silencer and reference system as detailed above, are shown in Figure 9. Once again, the time-invariant analysis with source impedance values given by the direct method gives very accurate results. This is quite surprising, since the pressure within the source region and the volume velocity out of the source region both vary with acoustic load in this example, in violation of the assumptions inherent in the time-invariant analysis. Of less surprise is that the inaccurate values for source impedance, as given by the direct method, yield accurate insertion loss results. Although the values are inaccurate, they are nevertheless very large, and effectively large enough to give the same reflective condition as the even larger exact values. Furthermore, since the values are accurate for all harmonics, it follows that the division of energy between the various harmonics in the exact analysis is effectively governed by the source condition and valve motion, not by the acoustic load. The

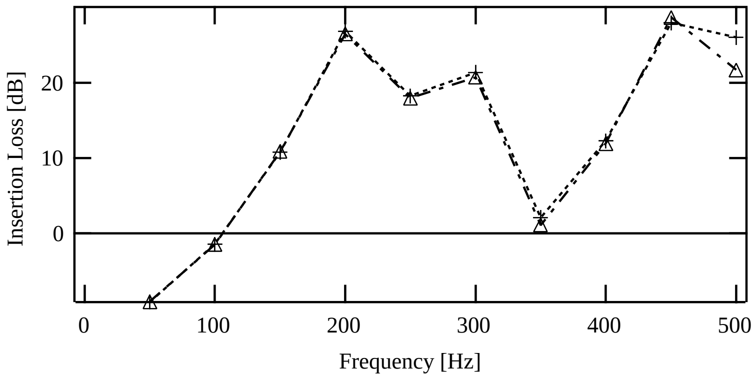

Figure 9. Insertion loss for a source of constant piston velocity: ------, + actual; ------, Δ from "measured" source impedance.

noticeable inaccuracies in insertion loss values at the highest harmonics can be discounted. Since only 10 harmonics were used in the analysis, one would expect some inaccuracies in the analysis for the ninth or tenth harmonic.

As noted earlier, there are sufficient secondary noise sources present in the exhaust system of an IC-engine for insertion loss results to be obtained throughout a frequency spectrum at a single operating speed of the engine. The analysis can be simply extended to model this case. A secondary broadband noise source of invariant volume velocity is assumed to exist just downstream of the valve. In a purely linear analysis, such as is assumed here, there is no interaction between the sound fields of the primary engine source and the secondary source, when the frequency of the secondary source does not equate to any of the harmonics of the valve and piston motion. In such cases, the source is effectively a constant volume region, as in the analysis of impedance by the direct method. Thus, from equations (6) and (10)

$$([\mathbf{D}] - [\mathbf{A}][\mathbf{C}])\{\mathbf{v}\} = -[\mathbf{A}][\mathbf{Z}_L]\{\mathbf{v} + \mathbf{v}_I\}, \tag{23}$$

where $\{\mathbf{v}_I\}$ is the volume velocity vector of the secondary source. For a given acoustic load and secondary source, equation (23) yields the volume velocity through the valve. The source pressure and ultimately insertion loss of two separate systems then follow as before.

At harmonic frequencies of the piston motion, it is assumed that the primary source dominates the secondary source, such that the latter is negligible. The insertion loss is then given by the preceding method, for a source of invariant piston velocity. Figure 10 shows a complete spectrum of insertion loss from such an analysis, as compared with the insertion loss assuming a time-invariant source with an impedance as given by the direct method. There is no discernible difference between the two curves. The actual errors incurred when using the time-invariant analysis are of even smaller magnitude at frequencies where only the secondary source emits sound than at harmonic frequencies of the piston and valve motion. A simple time-invariant model with an assumed infinite source impedance would give the same insertion loss spectrum.

6. CONCLUSIONS

An analytical model has been presented of the direct method of measurement of the effective source impedance of a linear time-variant source. The use of an effective frequency-dependent source impedance cannot adequately represent the actual

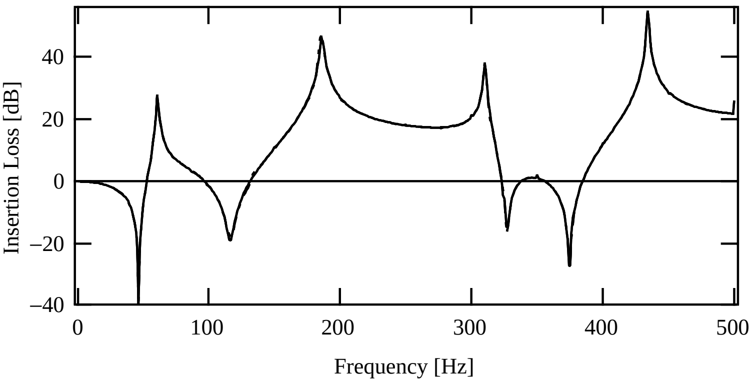

Figure 10. Insertion loss assuming secondary noise sources: ——, actual; – – – – – – –, from "measured" source impedance.

characteristics of a time-variant source. Indeed, the impedance values given by the direct method are theoretically functions both of the acoustic load and the location of the injected signal as used in the measurement process. However, in practice the influence of these factors has been found to be of little significance. Furthermore, the direct method yields physically realistic results, in that the source resistance is always positive, and the impedance values are at least large, even though they do not tend to infinity in the same manner as the coefficients of the impedance matrix of the actual source. The net effect is that the results for insertion loss with a time-variant source are predicted very accurately when using the effective source impedance in a time-invariant analysis. This observation remains true even when the strength of the source is allowed to vary with acoustic load.

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